

1. Gear Teeth

Clocks and Ships

In October, 1707 seamen on the British ship "Association", commanded by Admiral Clowdisley Shovel, miscalculated their longitude and ran aground on the Scilly Islands, off the far southwest corner of England. Eight hundred sailors, including Shovel, were lost.

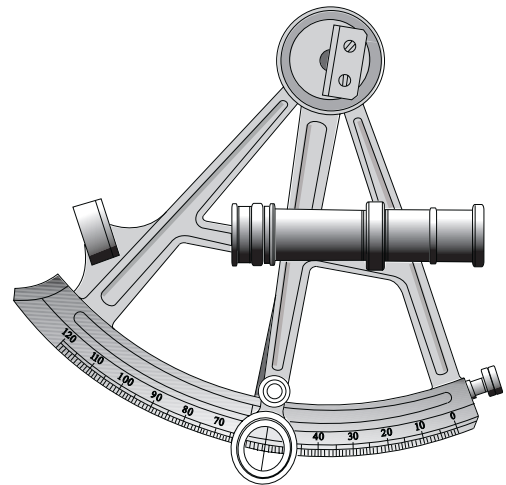
In 1707, longitude, or east-west position, was calculated by observing the sun at a certain time of day. The determination required an accurate measure of the sun's angle in the sky, using an instrument called a sextant, and accurate time-keeping. Before setting sail, a clock would be set to time of day at Greenwich, London, England, which is the arbitrary point of zero longitude. At any time during the journey, the clock had to show Greenwich time, not local time. Since even the best clocks at the time were inaccurate, however, disastrous mistakes in navigation, like Clowdisley Shovel's, were bound to occur.

In 1714, the British government offered a prize of twenty thousand pounds sterling to anyone who could figure out a way to measure longitude correct to within thirty nautical miles, after a voyage of six weeks.

The most accurate clocks at that time were pendulum clocks, which lost accuracy when ships pitched and rolled on the waves. An Englishman named John Harrison began building spring powered clocks for shipboard use.

In 1761, after thirty years of development, one of Harrison's clocks on the British ship "Deptford" sailed from England to Jamaica with an error in longitude measurement of well less than thirty miles.

It took the British government eleven years, but they finally paid Harrison his twenty thousand pound reward in 1773. He died three years later.



Sextant

The instrument used by seamen to measure the sun's angle in the sky. This measurement, plus an accurate clock showing Greenwich time, allows a seaman to calculate his east-west, and north-south position.

Introduction

For gears to run quietly and smoothly, the gear teeth must transmit **constant angular velocity**. This section tells us that constant angular velocity means the surface speed of the driven gear must equal the surface speed of the driver for every position of the teeth as they go in and out of mesh. We learn that many tooth shapes will transmit constant angular velocity, but all shapes except one need the **center distance (CD)** to be exact.

The one shape that works for varying center distance is the **involute curve** which can be generated by unwrapping a taut string from a cylinder. The end of the string draws an involute curve. On spur or helical gears, involute curves form the tooth flanks, which roll and slide together when the teeth mesh.

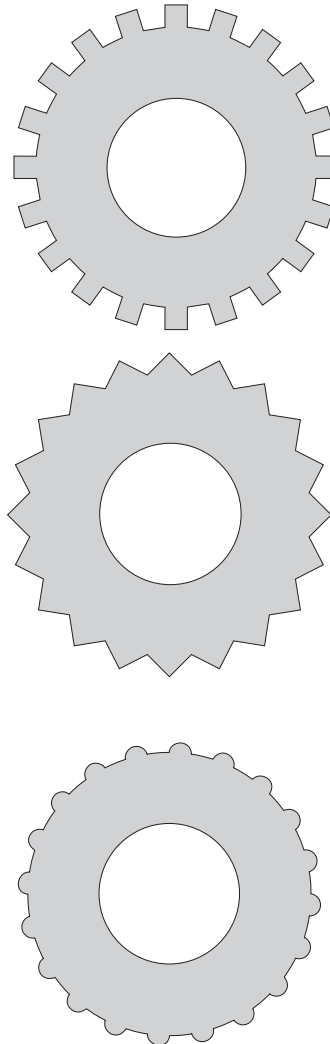
Root diameter is the bottom of a tooth space, **outside diameter (OD)** is the top of a tooth, **pitch diameter (PD)** is about halfway up the tooth height. **Base diameter** is the imaginary circle from which the taut involute generating string would be unwound. **Pressure angle** is the slant of the gear tooth flank at a particular point. The **line of action** is a line along which the gear teeth contact each other as a tooth goes in and out of mesh. Points of contact lie along the line of action.

Clearance between gear teeth in mesh is called **backlash**, and depends on the thickness of the teeth and the center distance.

A tooth is **in mesh** from the point where its outside diameter crosses the line of action, to the point where the mating gear outside diameter crosses the line of action. The segment of the line of action where teeth are engaged is called the **line of contact**. **Contact ratio** is the ratio of the length of the line of contact to the circular pitch measured on the base circle.

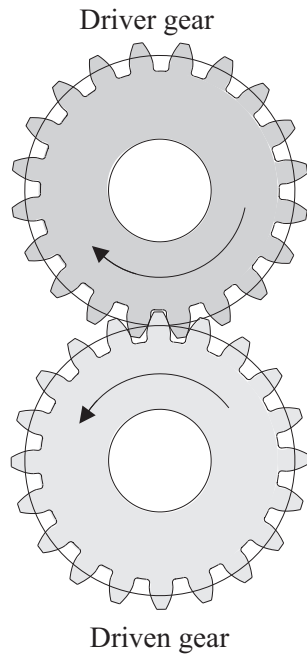
Constant Angular Velocity

Gear teeth can be made in almost any shape - triangular, square, curved.

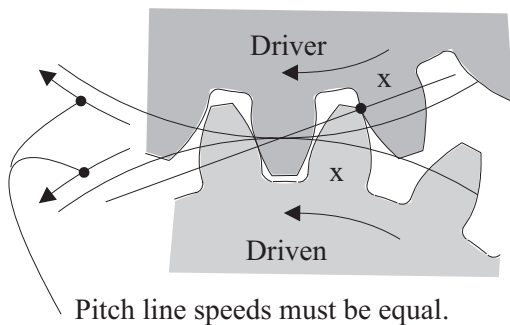


For gears to mesh smoothly, however, the teeth must be shaped so that they transmit constant angular velocity.

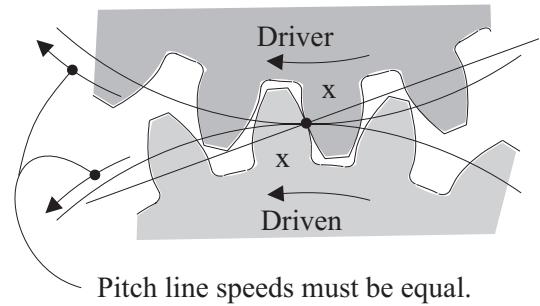
Constant angular velocity means that the driven gear follows the speed of the driver gear exactly. If the driver gear (the pinion) is turning at a constant speed, the driven gear must turn at a constant speed.



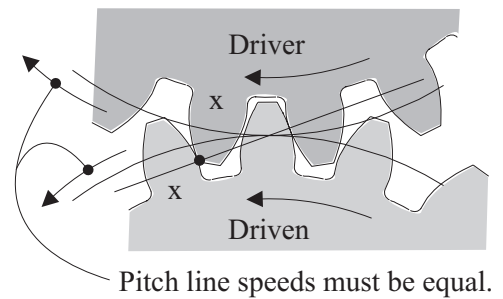
When a driven gear tooth is coming into mesh with a driver (pinion) tooth, it must follow the speed of the pinion exactly. Driven gear pitch circle speed, measured in feet per minute or meters per minute, must exactly equal the pitch circle speed of the pinion.



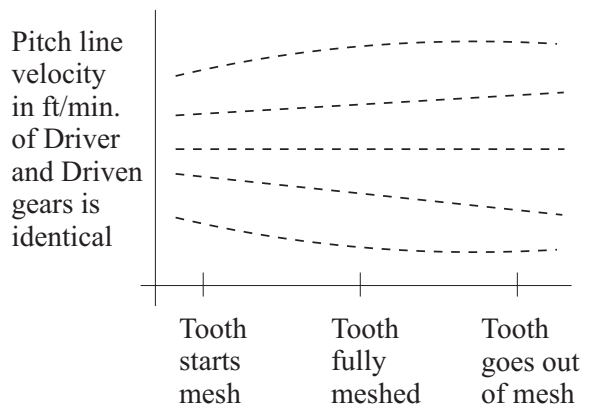
When the teeth are fully in mesh, the two pitch line speeds must remain exactly equal.



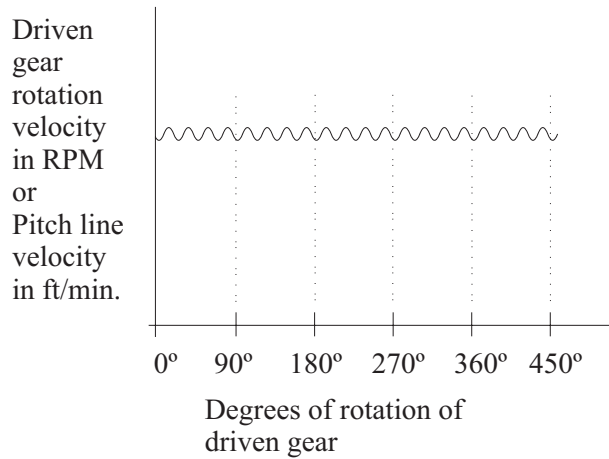
As the teeth go out of mesh, pitch line speeds must remain exactly equal.



When teeth transmit constant angular velocity, the pitch line velocity can stay steady, increase or decrease, but must be the same for Driver and Driven gear.



4. How many teeth do you think are in the driven gear which has a speed versus angular position graph as shown below?



Self Study Answers

Constant Angular Velocity

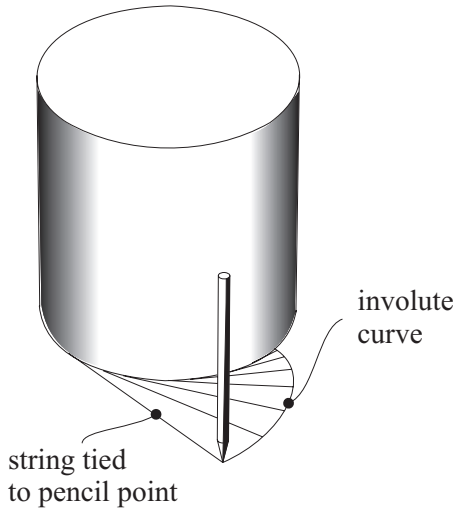
1. Transmission of constant angular velocity.
2. False. The driver pinion can change speed. Transmitting constant angular velocity means that the driven gear will exactly follow the pitch line speed of the driver as each tooth goes in and out of mesh. It's okay if the driver pinion changes RPM, as long as the pitch line speed of the driven gear follows, and does not vary as each tooth goes in and out of mesh.
3. The driven gear will speed up and slow down as each tooth goes in and out of mesh, causing pulsations (small changes) in speed which cause vibration and noise. The gears will not run smoothly or quietly.
4. The gear probably has 16 teeth. Each pulse or hump in the graph represents a tooth going in and out of mesh. One revolution of 360° would occur on the graph between 0° and 360° or between 90° and 450° . ($450 - 90 = 360$). Count the pulses on the graph between 0° and 360° or between 90° and 450° , and you find 16 pulses, caused by 16 teeth.

Involute Curve

There are many different tooth shapes that will transmit constant angular velocity. They almost all have one weakness, however. They almost all require the center distance between the gears to be held accurately to the calculated design dimension. If the center distance is not exactly correct, the teeth will not transmit constant angular velocity, and will cause pulsations and vibration.

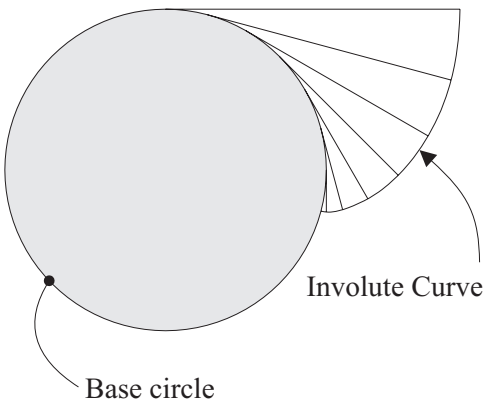
There is only one tooth shape that will transmit constant angular velocity even when the center distance varies, and that shape is called an **involute curve**.

Wrap a string around a water glass or other cylindrical object, tie a pencil to the loose end, pull it tight, and unwind it from the cylinder, keeping the string taut as you unwind it. The pencil will draw an involute curve as it's unwound.



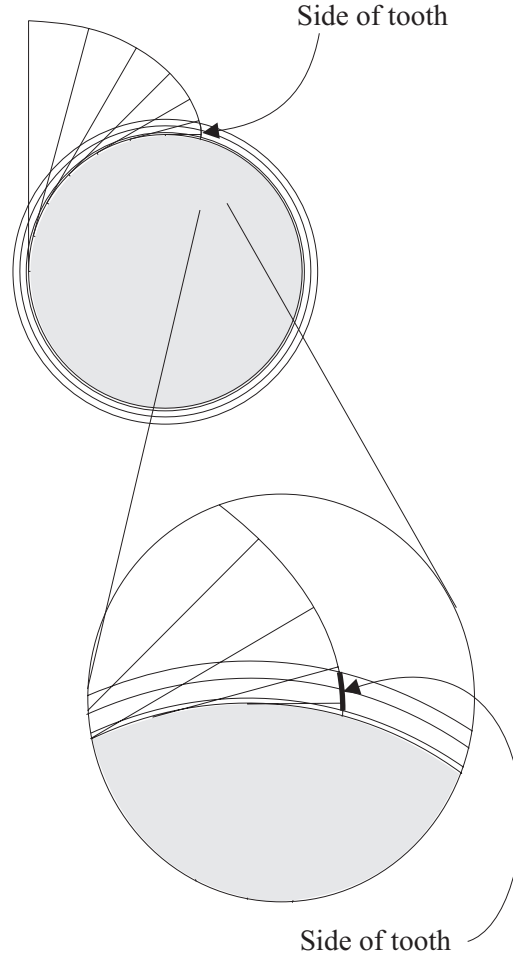
To be more precise, the curve drawn by the pencil should be called the **involute of a circle**, because there can be other involutes, drawn from ellipses or other curves.

Involutes used in gearing are involutes of a circle; when we use the word involute in this book, we mean involute of a circle.

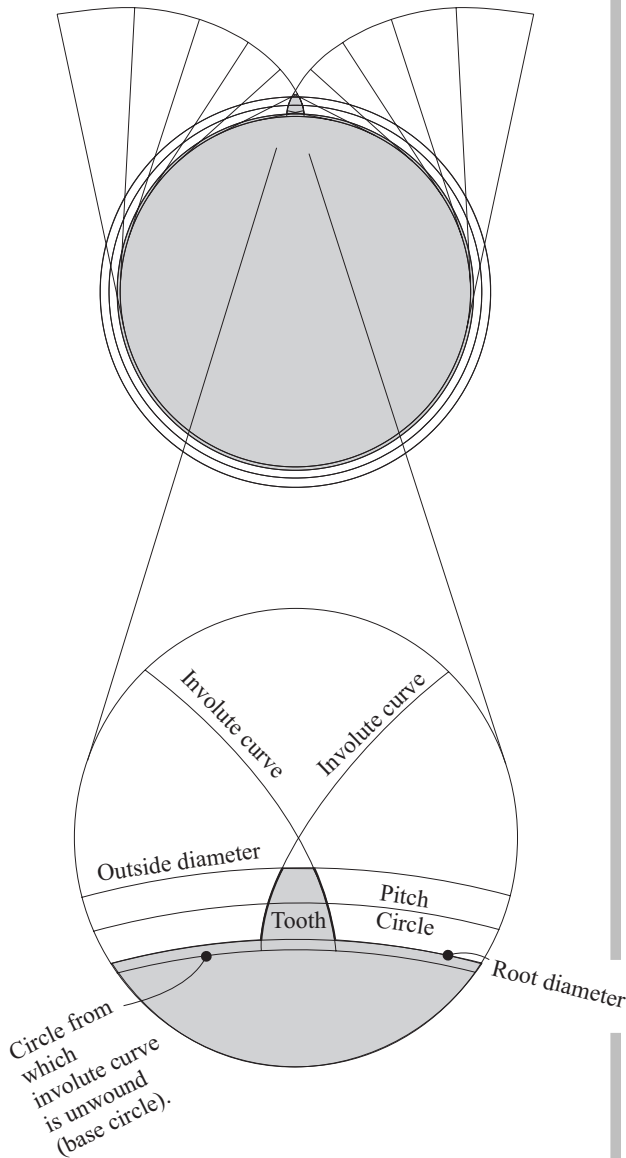


We call the circle from which the involute curve is unwound the **base circle**.

The side, or **flank**, of gear teeth follows the shape of the involute curve.



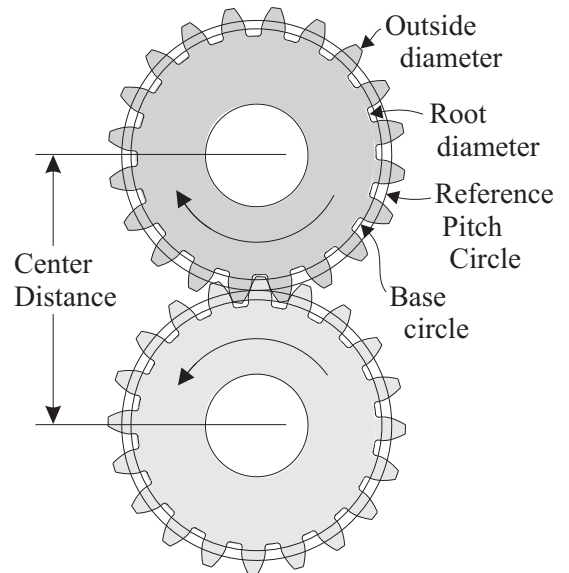
Each tooth has two involutes, one for the front side of the tooth, and one for the back.



The sides, or **flanks** of gear teeth are shaped as an involute curve. The bottom of a tooth space is the **root diameter**. The top of a tooth is the **outside diameter** of the gear.

The imaginary circle from which the involute is unwound is the **base circle**.

Reference Pitch Circle is an imaginary circle usually located approximately midway up the gear tooth.

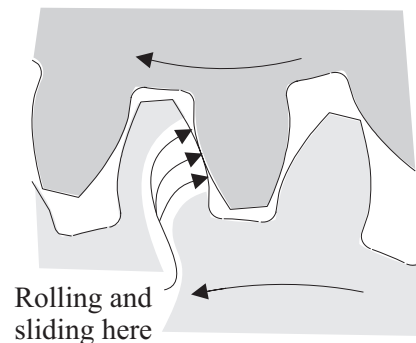


Pitch Circle diameter = **Pitch Diameter** = PD

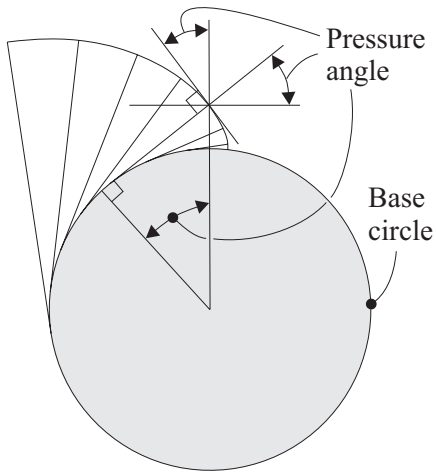
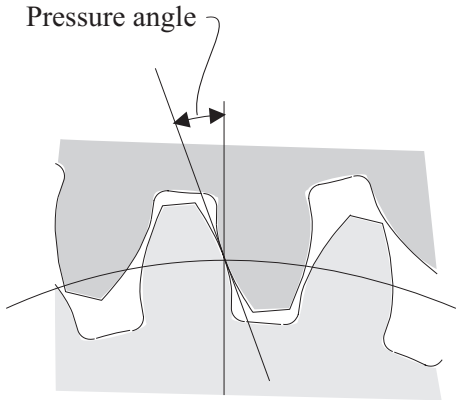
Center Distance (CD) is the distance between the centers of two mating gears.

$$CD = \frac{PDA + PDB}{2}$$

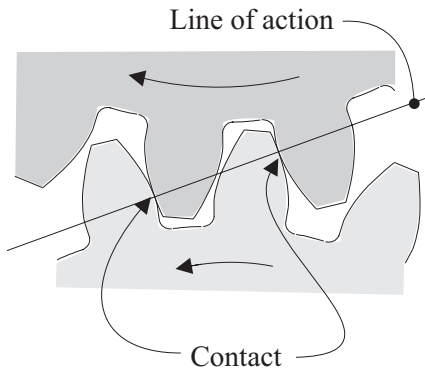
When two spur or helical gears mesh together, their involute curved tooth flanks roll and slide against each other.



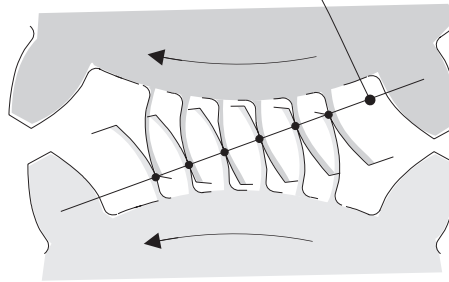
The angle of slant of the gear tooth flank at any point on the flank is called the **pressure angle**.



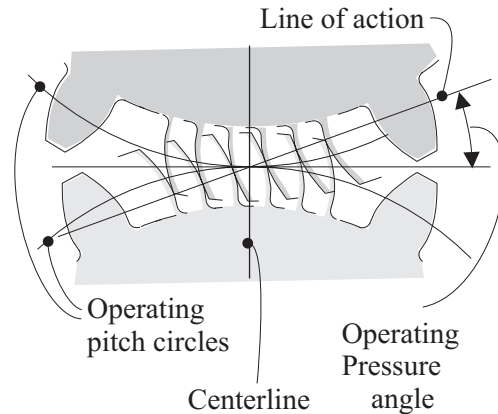
The **line of action** is a line along which the gear teeth contact each other as a tooth goes in and out of mesh.



Points of contact, the places in space where teeth touch, lie along the line of action.



For involute gear teeth, the line of action is a straight line, positioned at an angle to the centerline of the gears like this.



Two circles are tangent to each other, touching where they cross the centerline of the two gears. The circles are called **operating pitch circles**.

The line of action slants at an angle which is the **operating pressure angle**, the pressure angle of the teeth at the operating pitch circle.

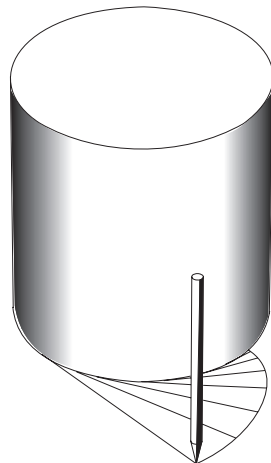
Circular pitch is the distance between centers of two adjacent teeth. For two involute gears to mesh properly, their circular pitches measured on the circumference of the base circle must be equal.

Self Study Questions Involute Curve

1. The name of the curve which forms a gear tooth flank is_____.
2. What is the name of the circle from which an involute curve is unwound?
3. If the front side of a gear tooth is shaped like an involute curve, what is the shape of the back side of the same tooth?
4. True or false?

Involute curves allow gear teeth to roll together smoothly with no sliding friction.

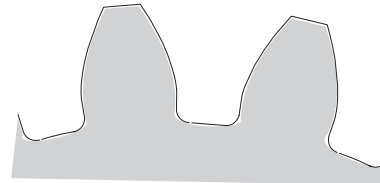
5. What is the angle of slant of a gear tooth flank called?
6. What is the name of the curve traced by the pencil?



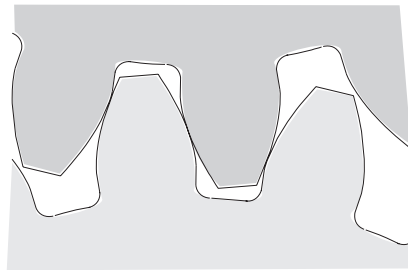
7. True or false?
 - a. Gears with involute curved teeth run smoothly because they transmit constant angular velocity.
 - b. If gears do not run on correct centers, points of contact will not be on the line of contact.
 - c. Line of action for involute gear teeth is an involute curve.

8. Imagine two involute spur gears running together with excessively large spaces, or clearance, between the teeth. Is it acceptable to reduce the spaces between teeth by bringing the gears closer together?

9. Draw arrows pointing to involute curves.

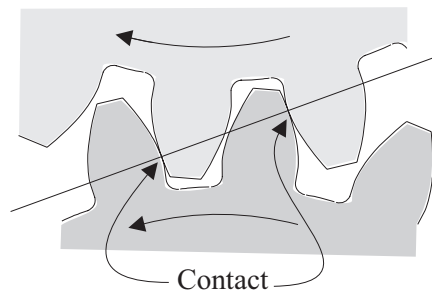


10. These two gears are in mesh with no backlash. Draw the line of action.



11. True or False?
Involute gear teeth roll together smoothly without sliding, in order to give constant angular velocity and smooth transmission of motion.

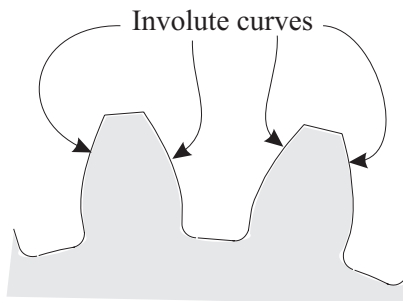
12. Mark the driver gear with an R, and the driven gear with an N.



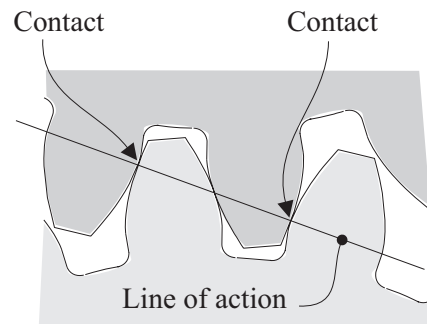
13. There is a gear tooth shape called cycloidal, which is fairly simple to describe, and transmits constant angular velocity throughout the complete gear mesh. Why do you think it is not used in modern gear teeth?

Self Study Answers Involute Curve

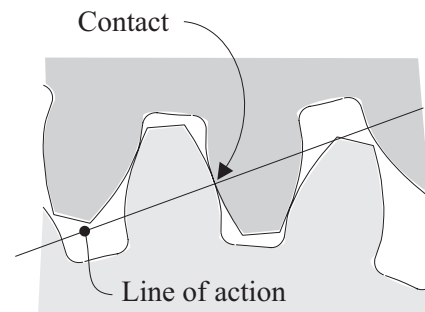
1. Involute curve
2. Base circle
3. Involute curve. Both sides of the tooth are involute curves.
4. False. The action between involute gear teeth is both rolling and sliding.
5. Pressure angle
6. Involute Curve
7. a. True
b. False
c. False. It is a straight line.
8. Yes, you can change center distance with involute spur gears and the involute teeth will still mesh smoothly. This ability to run smoothly on different centers is one of the main reasons that gear teeth are made in an involute form.
- 9.



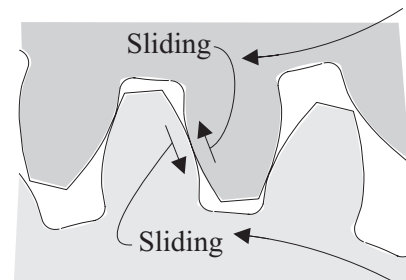
10. Line of action connects points where gear teeth touch.



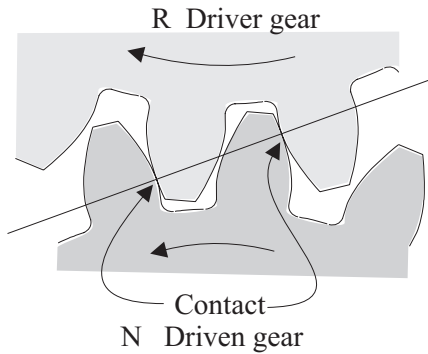
The line of action could also be like this, depending on the direction of gear rotation, and which gear is the driver.



11. False. Involute gear teeth roll together, but the teeth also slide against each other.



12.

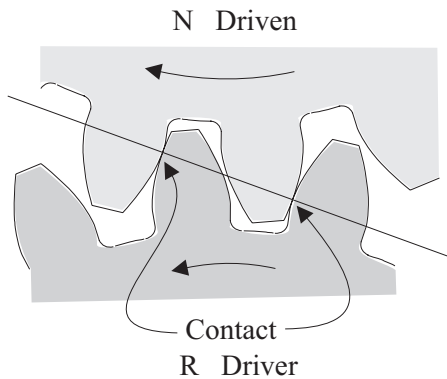


Notice that there is clearance between the meshing teeth, space between the right flank of the top gear and the left flank of the bottom. (This space is called **backlash**.)

The top gear must be the driver because its meshing teeth are moving to the left, and it contacts the mating driven gear with the left flank of its own tooth.

The bottom gear is moving to the left. If it were the driver, backlash would have to be on its right flank, the trailing flank, not its left.

If the bottom gear was the driver, the diagram would look like this:



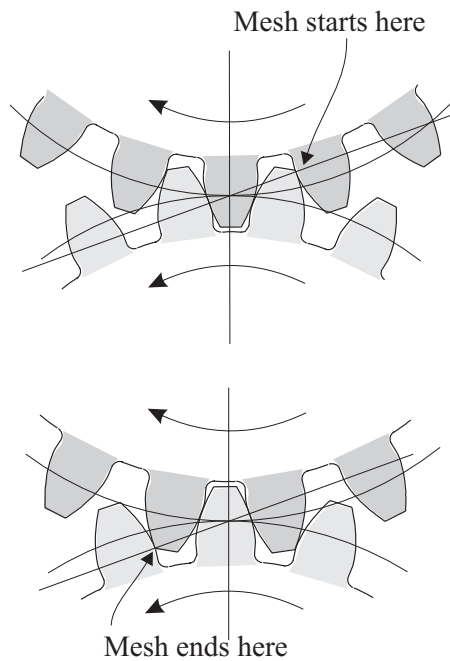
13. The cycloidal tooth form, like all other tooth forms except the involute, will not transmit constant angular velocity unless the gear center distance is held to exact design dimensions.

Contact Ratio

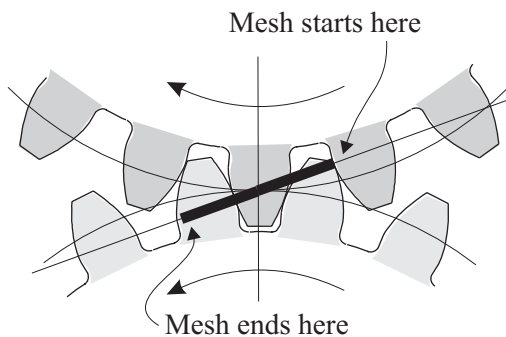
Before one tooth goes out of contact in a gear mesh, another tooth has to come into tooth contact, or else the gears won't work.

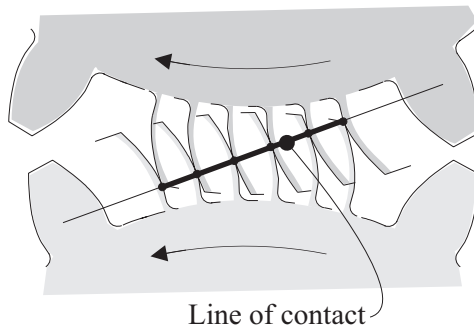
The gear teeth have to overlap.

A tooth is **in mesh** from the point where its outside diameter crosses the line of action, to the point where the mating gear outside diameter crosses the line of action.



The segment of the line of action where teeth are engaged is the **line of contact**.





The amount of overlap is measured by the **contact ratio**. Contact ratio is the ratio of the length of the line of contact to the circular pitch measured on the base circle.

If center distance is increased, contact ratio decreases. If the contact ratio becomes less than 1.0, the gears come out of mesh.

All things being equal, a pinion with a small number of teeth will mesh with a lower contact ratio than a pinion with a large number of teeth. The fewer teeth in a pinion, the lower the contact ratio.

Self Study Questions Contact Ratio

1. The length of the line of contact starts where two lines cross. What are the two lines?
2. What happens to the length of the line of contact when center distance increases?
3. What happens if the contact ratio is less than 1.0?
4. What is space between driver and driven gear teeth called?

Self Study Answers Contact Ratio

1. The outside diameter of the gear and the line of action.
2. It shortens.
3. The gears come out of mesh. A contact ratio less than one means there is no overlap in the tooth mesh, and the first tooth comes out of mesh before the second comes into mesh.
4. Backlash.

Summary

In this section we learned that for gears to run quietly and smoothly, the teeth must transmit **constant angular velocity**. Constant angular velocity means the surface speed of the driven gear must equal the surface speed of the driver for every position of the teeth as they go in and out of mesh. If **transmitted velocity** is not constant, pulsations and vibration will occur. We learned that many tooth shapes will transmit constant angular velocity, but all shapes except one need center distance to be exact.

The one shape that works for varying center distance is the **involute curve**. An involute curve can be generated by unwrapping a taut string from a cylinder. The end of the string draws an involute curve. On a gear, the involute curve forms the tooth **flank**. When spur or helical gears mesh, their teeth roll and slide together.

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Clearance between gear teeth in mesh is called **backlash**, and depends on the thickness of the teeth and the center distance.

A tooth is **in mesh** from the point where its outside diameter crosses the line of action, to the point where the mating gear outside diameter crosses the line of action. The segment of the line of action where teeth are engaged is the **line of contact**. **Contact ratio** is the ratio of the length of the line of contact to the circular pitch measured on the base circle.

In the next section, we find that **linear surface speed** of a rotating gear is proportional to diameter. **Operating pitch circles** are the imaginary circles on two mating gears where the linear surface speed of the circles are equal.

Circular pitch and **pressure angle** depend on the diameter at which they are measured. The **reference pitch circle** is an imaginary circle where the circle diameter, tooth pressure angle, and tooth circular pitch are all known.

Reference dimensions and **base circle diameter** are built into a gear when it is manufactured, and never change. **Operating dimensions** depend on the **center distance**, and change if it is changed. When people specify gear teeth, diametral pitch or module, and pressure angle, they usually mean reference dimensions, not operating dimensions.

Operating pitch diameter and **operating pressure angle** are not affected by tooth thickness, which only affects backlash.

Base circles of two gears in mesh can be imagined to be two spools. Imagine a string unwinding from one spool onto the other. The spools are the base circles, the string is the line of action, the operating pressure angle is the slant of the string.

The **line of action** lies tangent to the base circle of each gear, at an angle equal to the operating pressure angle.

The **operating pressure angle** gets larger when gears are moved apart, smaller when they are moved closer.

Base circle diameter equals reference pitch diameter times cosine reference pressure angle.

